Elementary Geometry
Session 4

Topic	Activity Name	Page Number	Related SOL	Activity Sheets	Materials
Transformational Geometry: Tessellations	Sums of the Measures of Angles of Triangles	103	5.13		Paper, scissors, and rulers
	Do Congruent Triangles Tessellate?	106	3.18, 4.17, 5.13, 5.14	Types of Triangles, Sum of the Measures of the Angles, Tessellations of Triangles 1 and 2, Congruent Scalene Triangles	Paper, scissors, and rulers
	Do Congruent Quadrilaterals Tessellate?	113	4.17, 5.15	Sum of the Measures of the Angles, Tessellations of Quadrilaterals 1, 2, and 3	Paper, scissors, and rulers
	Tessellations by Translation	118	4.17, 5.15	Tessellations by Translation, Square	1 large square piece of paper per participant, rulers and scissors
	Tessellations by Rotation	121	4.17, 5.15	Tessellations by Rotation	1 large square piece of paper per participant, rulers and scissors
Solid Geometry	Solid Figure Sort	124	2.20, 2.22, 3.18, 4.17, 5.16		1 set of geometric solids per group of 4-6
	What's My Figure? Ask Me About It.	125	2.20, 2.22, 3.18, 4.17, 5.16		1 set of geometric solids per group of 4-6
	What's My Figure? Touch Me.	126	2.20, 2.22, 3.18, 4.17, 5.16		1 set of geometric solids per group of 4-6
	Take It Apart	127	2.20, 2.22, 3.18, 4.17, 5.16		Cardboard cereal boxes, canisters, milk cartons, scissors
	Building Solid Figures	128	2.20, 2.22, 3.18, 4.17, 5.16		Scissors and tape or glue; or solid figures, D-stix, or other commercial 3-dimensional building kit

Topic: Transformational Geometry: Tessellations

Description: Patterns of geometric design are all around us. We see them every day, woven into the fabric of the clothes we wear, laid underfoot in the hallways of the buildings where we work, and printed on the wallpaper of our homes. Whether simple or intricate, such patterns are intriguing to the eye. We will explore a special class of geometric patterns called *tessellations*. Our investigation will interweave concepts basic to art, to geometry, and to design.

The word *tessellation* means, "A design that completely covers the surface with a pattern of figures with no gaps and no overlapping." It comes to us from the Latin *tessela*, which was the small, square stone, or tile used in ancient Roman mosaics. *Tilings* and *mosaics* are common synonyms for tessellations. Much like a Roman mosaic, a *plane tessellation* is a pattern made up of one or more figures, completely covering a surface without any gaps or overlaps. Note that both two-dimensional and three-dimensional figures will tessellate. Two-dimensional figures may tessellate a plane surface, while three-dimensional figures may tessellate space. We will use the word *tessellation* alone to always mean a plane tessellation.

Although the mathematics of tiling can become quite complex, the beauty and order of tessellation is accessible to anyone who is interested. To analyze tessellating patterns, you have to understand a few things about geometric figures and their properties - but all you need to know is easily explained in a few pages.

We will approach this subject through directed exploration. We will be exploring the following questions: Which figures will tessellate (that is, tile a plane without overlapping or leaving spaces)? Why will certain figures tessellate and others not? How many different tessellating patterns can we create using two or more regular two-dimensional plane figures? Do tessellating designs have symmetry? If so, what kind? How can we use transformations (slides, flips, and turns) to create unique tessellations? What other techniques could we use to generate the intricate designs?

In the twentieth century, a number of fine artists have applied the concept of tessellating patterns in their work. The best known of these is Dutch artist M. C. Escher. Inspired by the Moorish mosaic designs he saw during a visit to the Alhambra in Spain in the 1930s, Escher spent most of his life creating tessellations in the medium of woodcuts. He altered geometric tessellating figures into such forms as birds, reptiles, fish and people.

<u>Related SOL</u>: K.11, K.12, 1.16, 2.21, 3.18, 3.20, 4.17, 5.13, 5.14

Activity: Sum of the Measures of Angles of a Triangle

Format: Large Group/Small group

Objective: Participants will demonstrate that the sum of the measures of the interior angles of any triangle equals 180°, by cutting three random triangles out of paper, numbering and tearing off their vertices, and rearranging them adjacent to each other to form a straight angle.

Related SOL: 5.13

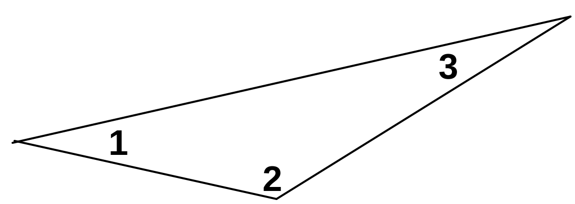
Materials: Paper, scissors, straightedges

Time required: Approximately 15 minutes

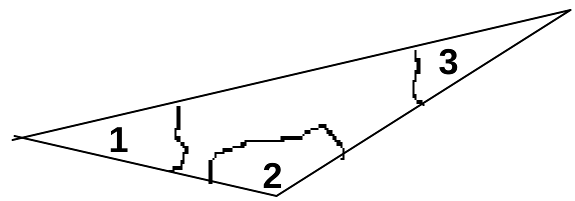
Directions:

1) Distribute the paper, scissors, and straightedges.

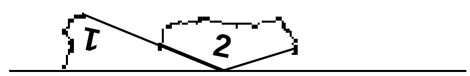
2) Have each participant draw 3 large triangles using a straightedge, and then cut the 3 triangles out. They should label the vertices of one triangle as 1, 2, and 3; label the vertices of the second triangles as 1*, 2*, and 3*; and label the vertices of the third triangle as 1', 2', and 3'.



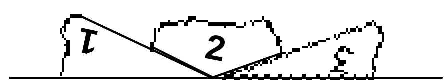
3. Have them tear off the corners. (That's right. Tear, not cut. By tearing, you can still determine which was the vertex. It will be the cut part.)



4. The participants should draw a dot on the page and a straight line through the dot. Have them place the cut vertex of <1 on the dot and the cut side on the line. They can trace the angle or tape it in place. They should choose another angle from the same figure and place its cut vertex on the dot, lining up the side of the first angle not on the line with a side of the second angle, and trace or tape it in place.



5. Have them repeat this sequence of steps until all the angles from one figure are adjacent to one another.



6. Ask what angle these combined corners form?

7. Repeat this procedure with the other two triangles. Ask if the same thing happens with all three triangles?

8. Have the groups share their results and then form conclusions focused on the following issues.
   a. Do the angles from all of their triangles always equal the same amount?
   b. Examine any triangle that turned out differently. Why do you think it did?
   c. Based on this activity, what conjecture can you make about the sum of the measure of angles in a triangle?

d. If you and your group members made 200 triangles, tore off the vertices, lined them up, and they all totaled the same, would this insure that the 201st triangle would come out the same?

e. If you could find a single triangle that came out differently, you would disprove the conjecture that the sum of the measures of the interior angles in a triangle is always 180°. Any item that doesn't fit your conjecture is called a **counterexample**. A single counterexample is enough to disprove a conjecture. Did you or any of your group members find a triangle whose angle measures didn't add up to 180°? Unfortunately, not finding one doesn't mean that one doesn't exist. However, it does give us more confidence in our conjecture.

Activity: Do Congruent Triangles Tessellate?

Format: Large Group/Small group

Objective: Participants will cut out sets of congruent triangles of various types and cover the plane with each type of triangle.

Related SOL: 3.18, 4.17, 5.13, 5.14

Materials: Paper, scissors, rulers, Types of Triangles Activity Sheet, Sum of the Measures of the Angles Activity Sheet, Tessellations of Triangles Activity Sheets 1 and 2, Congruent Scalene Triangles Activity Sheet

Time required: Approximately 20 minutes

Directions:

1) Remind the participants of the following definition: **Tessellation – a design that completely covers a surface with a pattern of figures with no gaps and no overlapping.**

2) Since a triangle is the simplest two-dimensional plane figure, we will start with the triangle in our investigation of which two-dimensional plane figures tessellate. Also, to keep things simple, have the participants explore tessellating with a single triangular figure rather than combinations of different triangular figures.

3) Review the definition of congruent triangles -- triangles that have the same size and figure.

4) Before proceeding with the investigation, have the participants look at some different types of triangles so that we can consider each type separately. Triangles are classified according to the relationships and the size of their sides and angles. Examples of each triangle type are shown on the Types of Triangles Activity Sheet.

5) Ask the participants to explore the following questions:

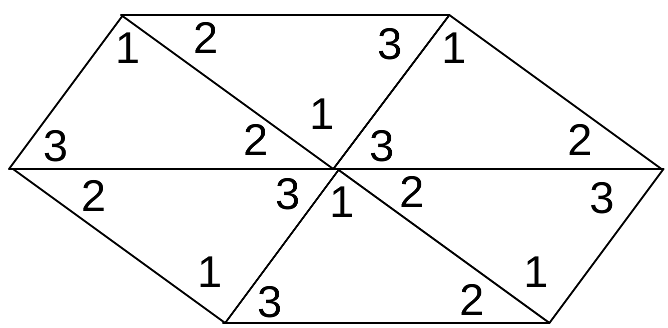
Which triangles (if any) tessellate?

If some triangles tessellate, do they all?

Start the investigation with scalene triangles. Have the participants draw a scalene triangle and copy it several times to make congruent triangles. They should label them so that all angle 1's are congruent, all angle 2's are congruent, and all angle 3's are congruent. The participants should cut the congruent triangles out and move them around to see if they tessellate. If you don't want the participants to create their own scalene triangles, you can just have them cut out several triangles from the Congruent Scalene Triangles Activity Sheet

6) Ask the participants if angles 1, 2, and 3 come together at a common vertex? What appears to be the sum of the measures of triangles 1, 2, and 3?

7) Using six congruent scalene triangles, we can completely fill all the space around the common vertex point of the six triangles. There are no gaps, no overlaps - a criterion of a tessellation. Do you think that all scalene triangles will tessellate in the plane? Why or why not? Discuss this question, referring to the Sum of the Measures of the Angles Activity Sheet.



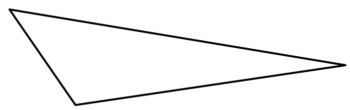
8) Try to make a tessellation using six congruent right triangles. Do you think that all right triangles will tessellate in the plane? Why or why not?

9) Can you make a tessellation using six congruent equilateral triangles or six congruent isosceles triangles?

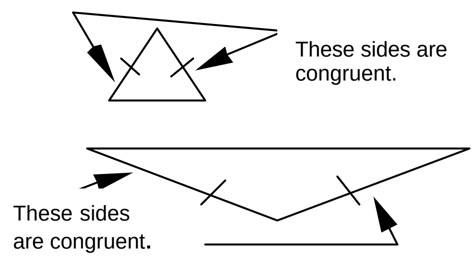
10) Examine the tessellations on the Tessellations of Triangles Activity Sheet. Identify the triangles used to form the tessellations. Do you think that any six congruent triangles can be used to make a tessellation? Why or why not?

Types of Triangles

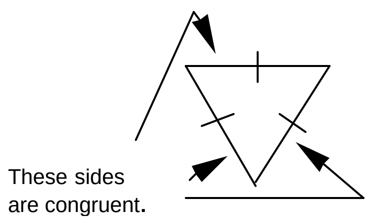
A **scalene** triangle has **no** congruent sides.



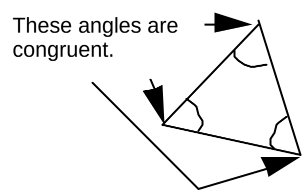
An **isosceles** triangle has **two or more sides**



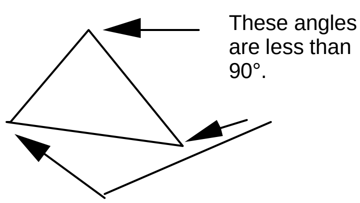
An **equilateral** triangle has all three sides congruent.



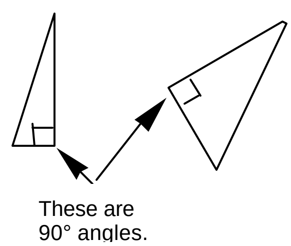
An **equiangular** triangle has all three angles congruent.



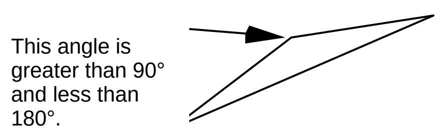
An **acute** triangle has all three angles acute.



A **right** triangle contains a right angle.

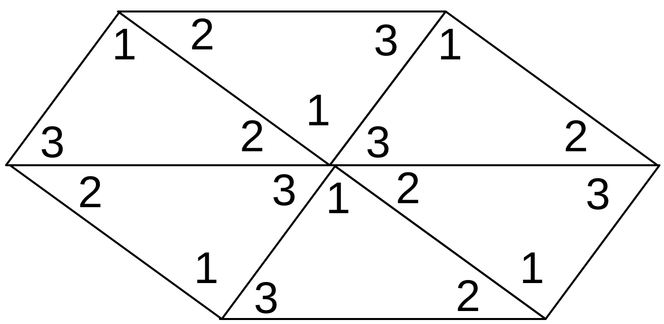


An **obtuse** triangle contains an obtuse angle.



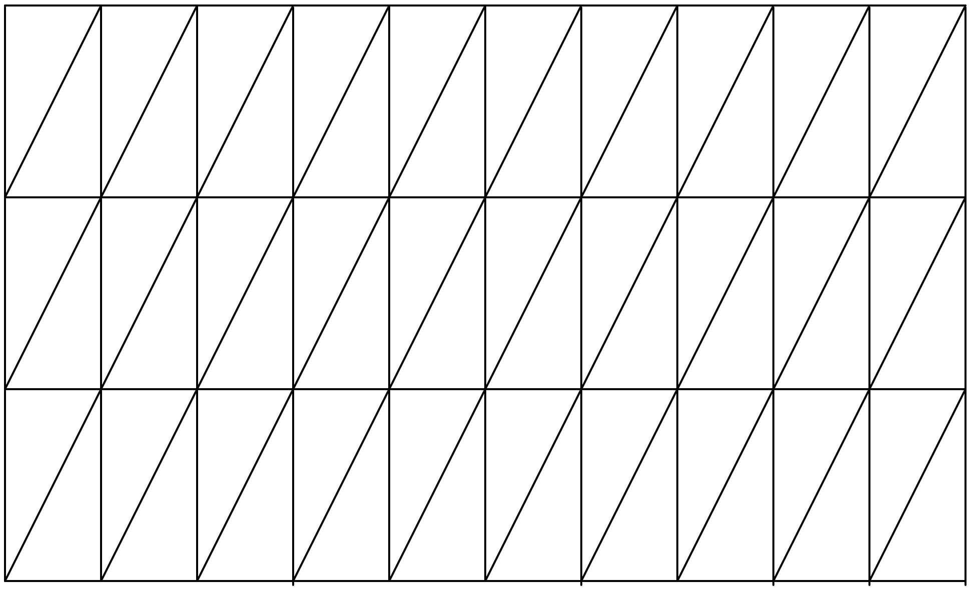
Sum of the Measures of the Angles

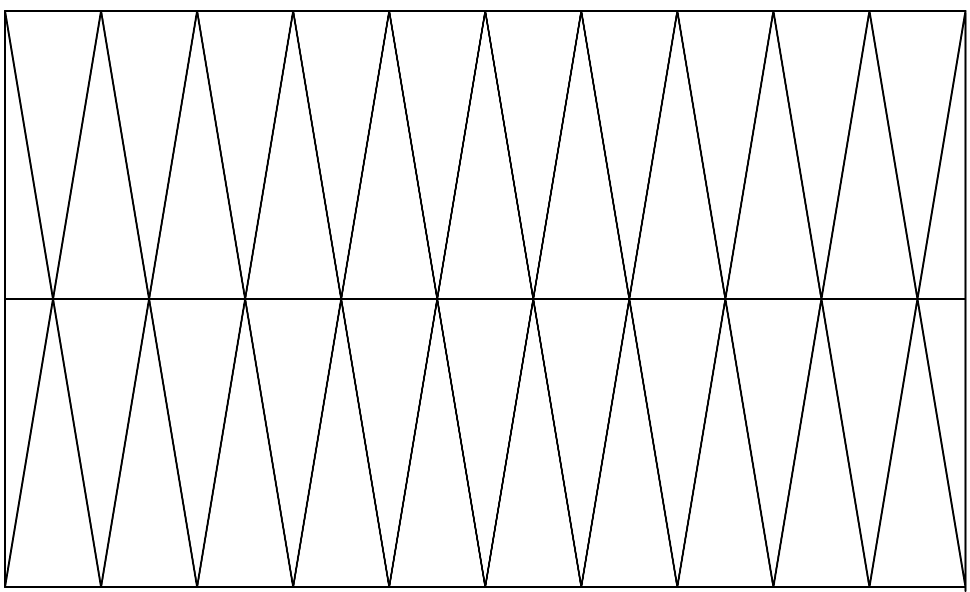
What is the sum of the measures of the angles about the center point?



Tessellations of Triangles

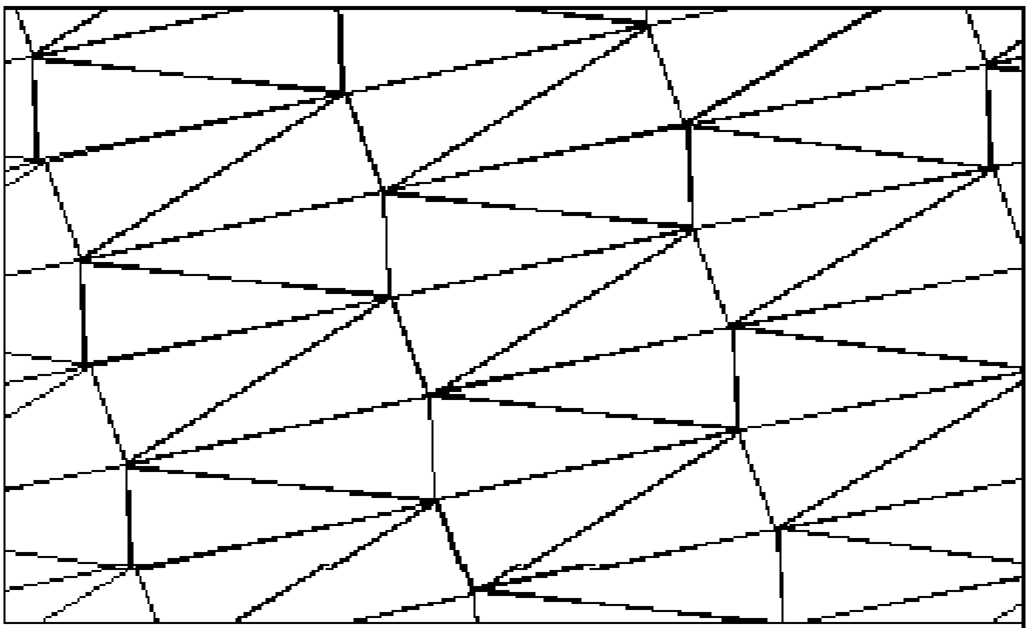
Page 1

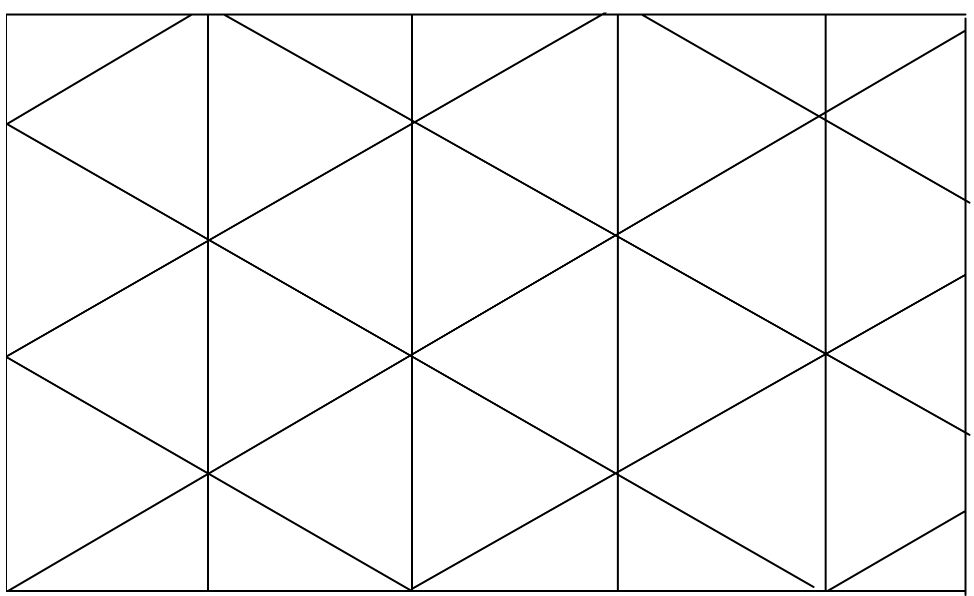




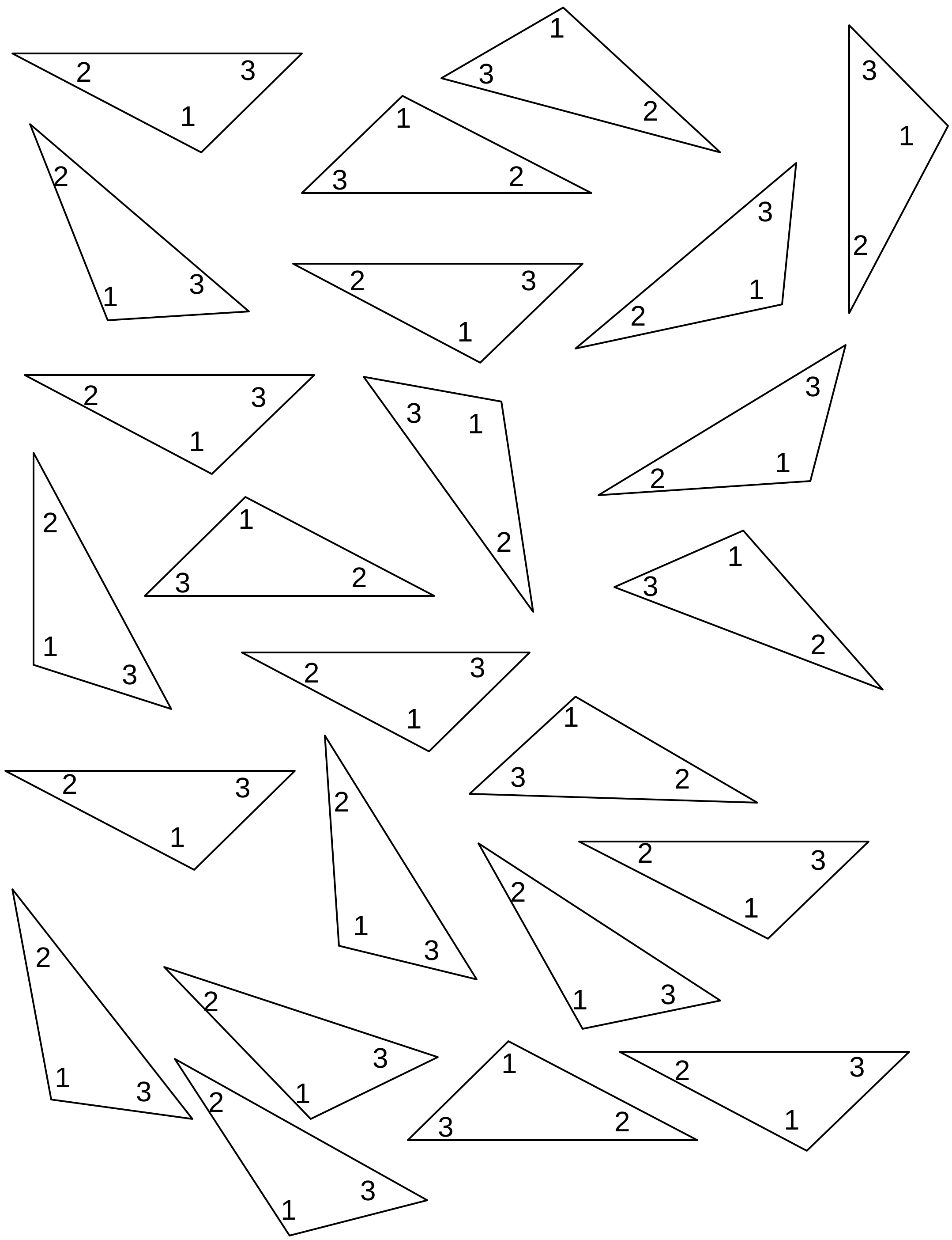
Tessellations of Triangles

Page 2





Congruent Scalene Triangles



Activity: Do Congruent Quadrilaterals Tessellate?

Format: Large Group/Small Group

Objective: Participants will determine whether congruent quadrilaterals tessellate.

Related SOL: 4.17, 5.15

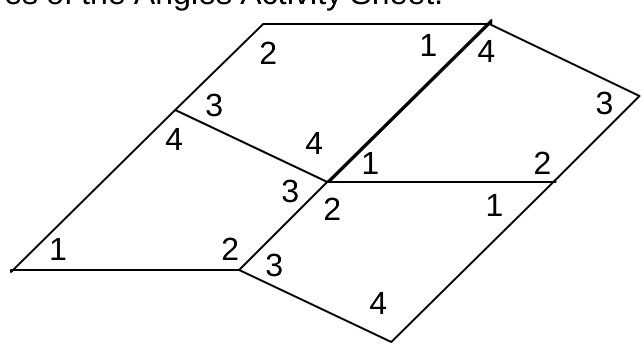
Materials: Paper, scissors, rulers, Sum of the Measures of the Angles Activity Sheet, Tessellations of Quadrilaterals Activity Sheets 1, 2, and 3

Time required: Approximately 15 minutes

Directions:

1) Repeat the previous activity entitled “Do Congruent Triangles Tessellate?” (page 106), using six or more congruent quadrilaterals (four-sided two-dimensional plane figures). Do your quadrilaterals tessellate?

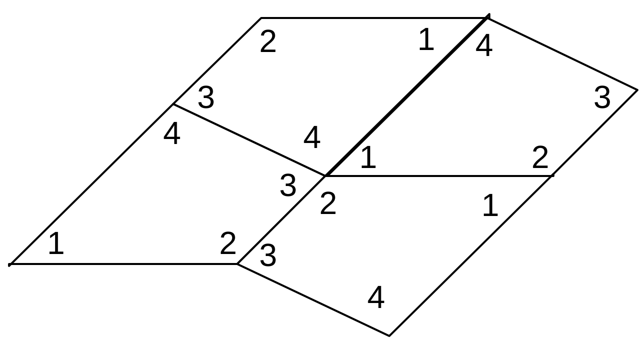
2) Using four congruent quadrilaterals, we can completely fill all the space around the common vertex point of the four quadrilaterals. There are no gaps, no overlaps - a criterion of a tessellation. Do you think that all quadrilaterals will tessellate in the plane? Why or why not? Discuss this question, referring to the Sum of the Measures of the Angles Activity Sheet.



3) Examine the tessellations in the Tessellations of Quadrilaterals Activity Sheets. Identify the quadrilaterals used to form the tessellations. Do you think that any four congruent quadrilaterals can be used to make a tessellation? Why or why not?

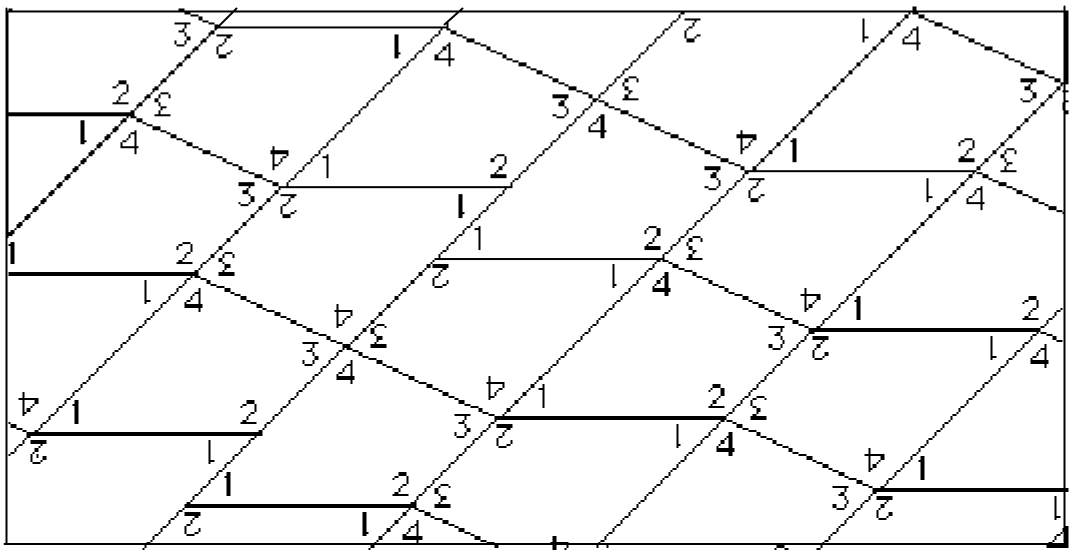
Sum of the Measures of the Angles

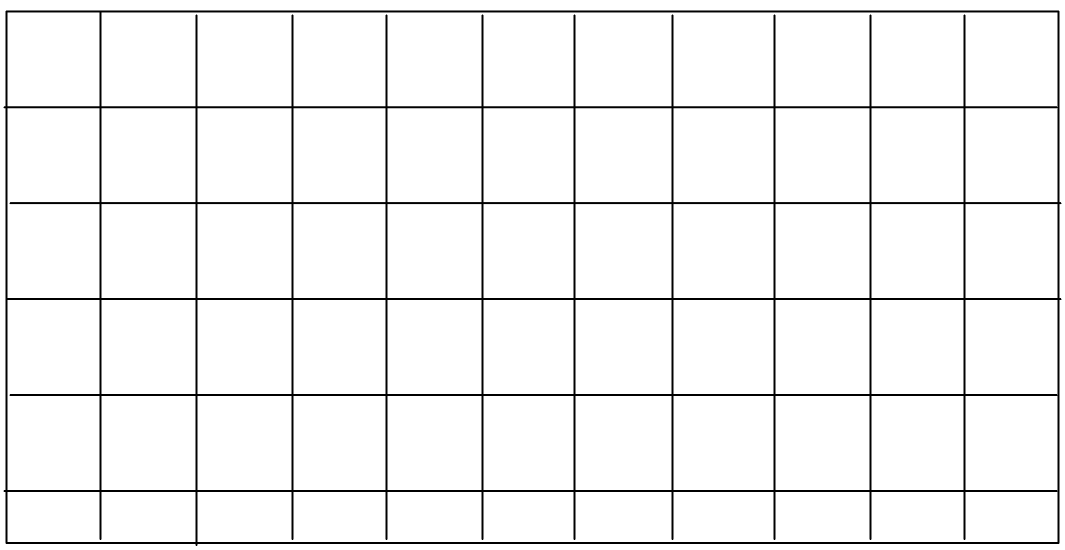
What is the sum of the measures of the angles about the center point?



Tessellations of Quadrilaterals

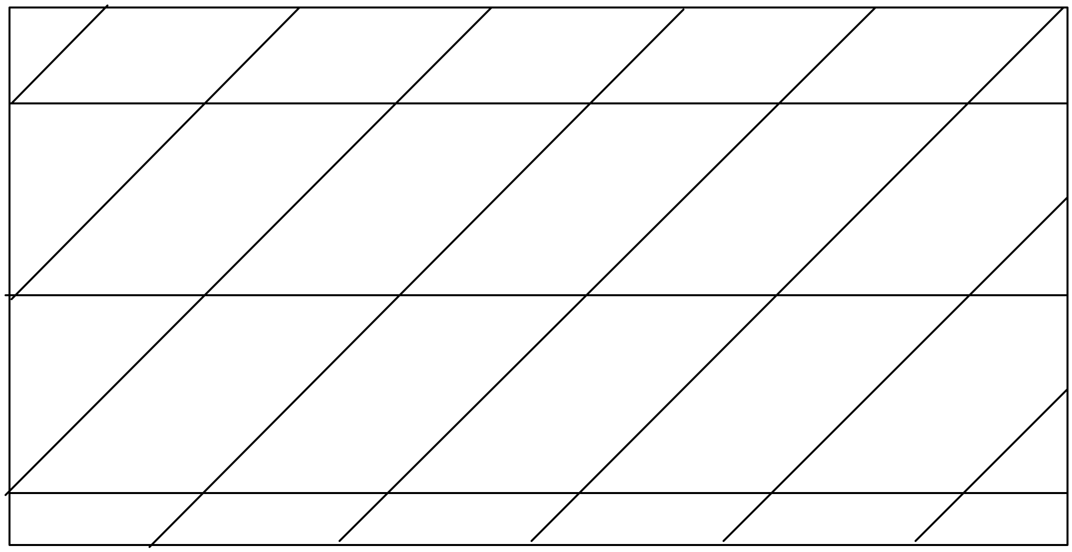
Page 1

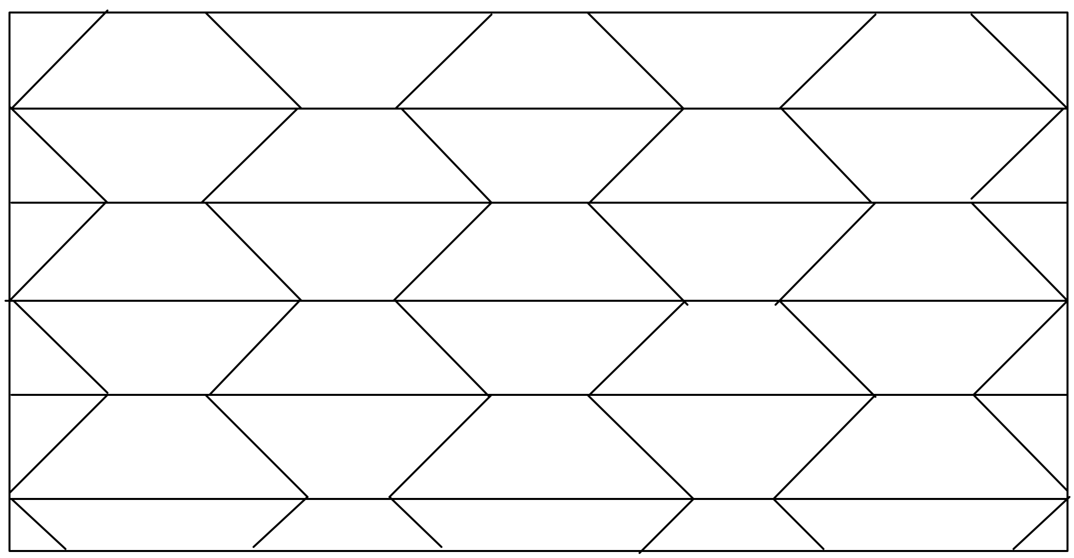




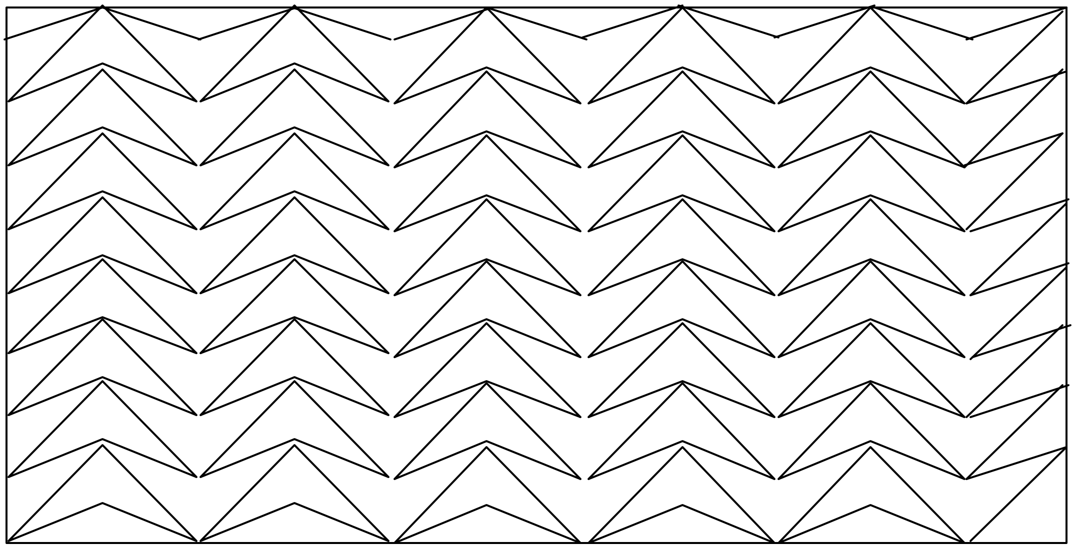
Tessellations of Quadrilaterals

Page 2





Tessellation of Quadrilaterals
Page 3



Activity: Tessellations by Translation

Format: Individual

Objective: Participants will create their own tessellations using translations.

Related SOL: 4.17, 5.15

Materials: One large square piece of paper, straightedge, scissors, Tessellations by Translation Activity Sheet, Square Activity Sheet

Time required: Approximately 40 minutes

Directions:

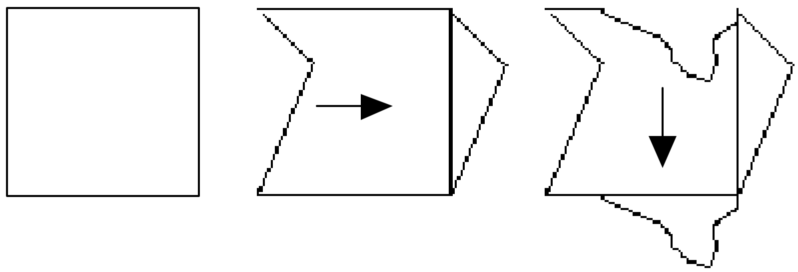
1) Sketch a figure extending into the square from one side on the Square Activity Sheet. Cut out that figure from the one side of the square and slide it across to the other side. Trace it as shown on the Tessellations by Translation Activity Sheet.

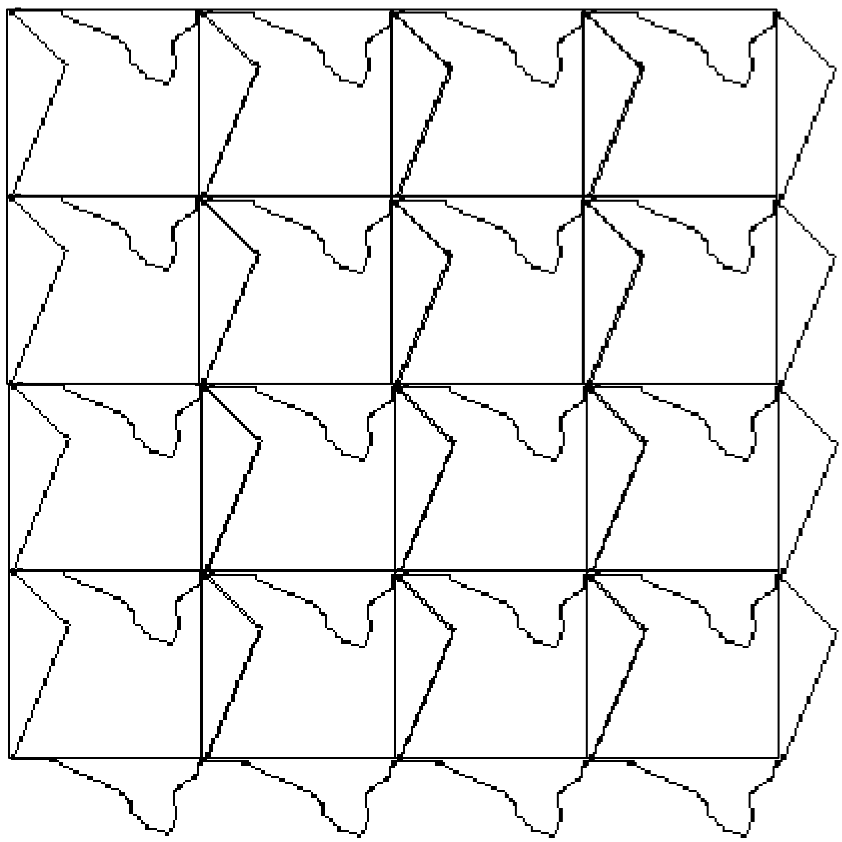
2) Sketch a figure extending into the square from the top on Handout 4.3. Cut out that figure from the top of the square and slide it down to the bottom. Trace it as shown on Tessellations by Translation Activity Sheet.

3) Cut out the other side and the bottom of the square along the figures you traced. Now you have your pattern to tessellate.

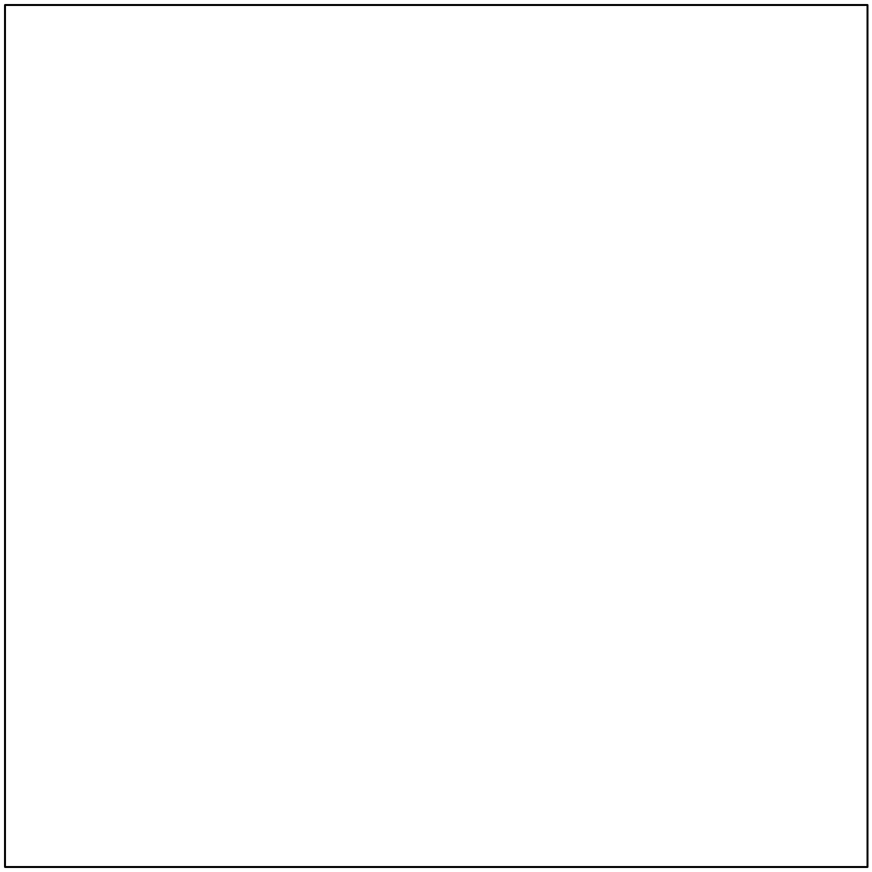
4) Trace the pattern repeatedly, translating it as shown on Tessellations by Translation Activity Sheet to form a tessellation.

Tessellations by Translation





Square



Activity: Tessellations by Rotation

Format: Individual

Objective: Participants will create their own tessellations using rotations.

Related SOL: 4.17, 5.15

Materials: One large square piece of paper, scissors, Tessellations by Rotation Activity Sheet, Square Activity Sheet (from previous activity)

Time required: Approximately 40 minutes

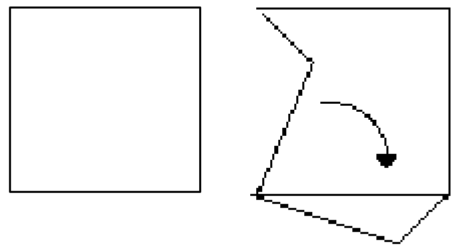
Directions:

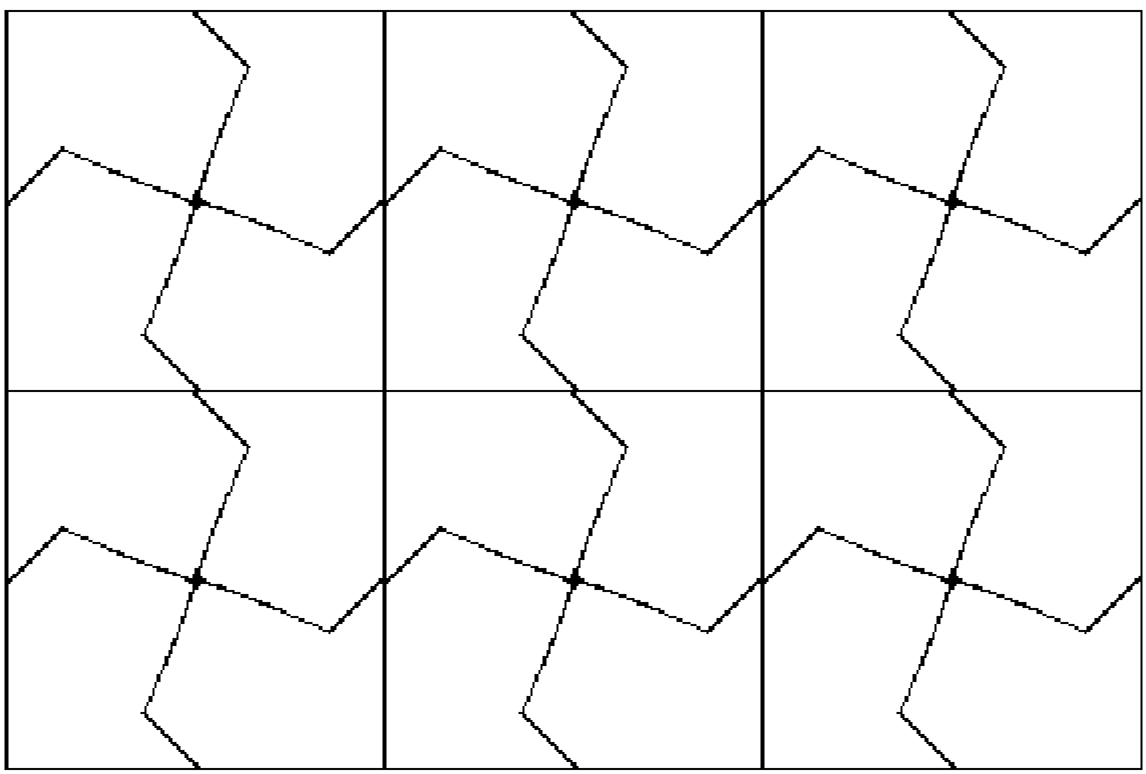
1) Sketch a figure extending into the square from one side on the Square Activity Sheet. Cut out that figure from the one side of the square and rotate it across to the other side. Trace it as shown on Tessellations by Rotation Activity Sheet.

2) Cut out the figure you traced and the other sides of the square. Now you have your pattern to tessellate.

3) Trace the pattern repeatedly, translating it as shown on Tessellations by Rotation Activity Sheet to form a tessellation.

Tessellations by Rotation





Topic: Solid Geometry

Description: Participants will explore three-dimensional figures by sorting and classifying them, determining what they are by touch, building them, and taking them apart.

Related SOL: 2.20, 2.22, 3.18, 4.17, 5.16

Activity: Solid Figure Sort

Format: Large Group/Small Group

Objectives: After performing their own sorts, participants will be able to distinguish the way students at various van Hiele levels of geometric understanding may sort solid figures.

Related SOL: 2.20, 2.22, 3.18, 4.17, 5.16

Materials: One set of geometric solids per group of 4-6

Time Required: Approximately 10 minutes

Directions:

1) Divide the participants into small groups. Distribute the sets of geometric solids, one set per group of 4-6 participants. Have them sort the solids into groups that belong together, sketching the pieces they put together and the criteria they used to sort. Have them sort two or three times, recording each sort.

2) Ask the participants to describe their sorts. Have them compare their sorts with those of other groups.

Activity: What's My Figure? Ask Me About It.

Format: Large Group

Objectives: Participants will use logical reasoning to determine the solid figure in the box after asking questions and receiving information about it.

Related SOL: 2.20, 2.22, 3.18, 4.17, 5.16

Materials: One set of geometric solids, box or bag

Time Required: Approximately 10 minutes

Directions:

1) The trainer says to the participants, "This box (or bag) contains a solid figure." Shake it so the participants can hear. "I'd like you to ask me some questions that can be answered with yes or no to figure out what is in the box."

2) Questions and answers continue until the participants can figure out what solid figure is in the box.

Activity: What's My Figure? Touch Me.

Format: Large Group

Objectives: Participants will determine the solid figure in the bag by touch alone.

Related SOL: 2.20, 2.22, 3.18, 4.17, 5.16

Materials: One set of geometric solids, a paper bag

Time Required: Approximately 10 minutes

Directions:

1) The trainer says to the participants, "This bag contains a solid figure." Shake it so the participants can hear. "One of you at a time may put your hand into the bag and touch the solid. Try to figure out what is in the bag."

2) The participants take turns touching the solid figure in the bag without looking and try to determine the solid figure.

Activity: Take It Apart

Format: Large Group/Small Group

Objectives: Participants will determine the figures that form various solid figures and analyze how they fit together to form the solid figures.

Related SOL: 2.20, 2.22, 3.18, 4.17, 5.16

Materials: Cardboard cereal boxes, canisters, milk cartons, etc., emptied and cleaned, scissors

Time Required: Approximately 10 minutes

Directions:

1) The trainer or group collects a variety of cardboard containers such as cereal boxes. The participants carefully cut apart containers along their seams, in such a way that they can be flattened out, but each piece is connected. This figure is called the **net**.

2) The group should identify each figure formed.

3) The group should see how many different nets they can find for the same container.

Activity: Building Solid Figures

Format: Large Group/Small Group

Objectives: Participants will determine the plane figures that form various solid figures and analyze how they fit together to form the solid figures.

Related SOL: 2.20, 2.22, 3.18, 4.17, 5.16

Materials: Scissors, tape or glue, geometric solids, and handout made by tracing the sides of various geometric solids; or Polydrons, D-stix, or other commercial three-dimensional building kit

Time Required: Approximately 20 minutes

Directions:

1) Choose a geometric solid from the set. Make a handout by tracing each of the faces to form its net. Repeat for two or three more solids.

2) Distribute scissors, tape or glue, and multiple copies of handout; or Polydrons, D-stix, or other commercial three-dimensional building kit.

2) Working in small groups, the participants should predict which solid the net will make. The trainer may show several solids, one of which is the one that the net will form.

3) After the predictions are made, the participants should cut out the net and tape it together.

4) The participants should compare their solids to their predictions and to the original solids.